[Maximum mark: 6]
The system of equations given below represents three planes in space.

$$x + 5z = 2$$

$$-2x + y - 6z = -1$$

$$2y + 8z = 6$$

1.
Show that this system of equations has an infinite number of solutions. [3]
2.
Find the parametric equations of the line of intersection of the three planes. [3]

[Maximum mark: 5]
The system of equations given below represents three planes in space.

$$x + 2y - 2z = 4$$

$$3x + 5y - 4z = 9 \qquad [\text{where } a, b \in \mathbb{R}]$$

$$4x + 6y + az = b$$

Find the set of values of a and b such that the three planes have no points of intersection.

[Maximum mark: 6]
The system of equations given below represents three planes in space.

$$x - 3y + 2z = 5$$
$$3x + 5y + az = b \qquad [\text{where } a, b \in \mathbb{R}]$$
$$4x + 2y - 3z = 7$$

Find the set of values of a and b such that the three planes have exactly one intersection point.

[Maximum mark: 9]
Consider the following system of equations:

$$x - 3z = -2$$
$$-3x + y + 6z = 3$$
$$2x - 2y + (a-4)z = b - 3$$

where $a, b \in \mathbb{R}$.

1. Find conditions on a and b for which
   1. the system has no solutions;
   2. the system has only one solution;
   3. the system has an infinite number of solutions. [6]
2. In the case where the number of solutions is infinite, find the general solution of the system of equations in Cartesian form. [3]

[Maximum mark: 9]
Consider the following system of equations:

$$x + y + 4z = 1$$

$$3x + 2y + 16z = 5$$

$$4x + 2y + (a - 1)z = b - 4$$

where $a, b \in \mathbb{R}$.

1. Find conditions on a and b for which
    1. the system has no solutions;
    2. the system has only one solution;
    3. the system has an infinite number of solutions. [6]
2. In the case where the number of solutions is infinite, find the general solution of the system of equations in Cartesian form. [3]

[Maximum mark: 8]
Consider the following system of equations:

$$x + y + z = -1$$

$$4x + 2y + z = 3$$

$$9x + 3y = p$$

where $p \in \mathbb{R}$.

1. Show that this system does not have a unique solution for any value of p. [4]
2. 1. Determine the value of p for which the system is consistent.
   2. For this value of p, find the general solution of the system. [4]

[Maximum mark: 11]
Consider three planes represented by the following system of equations:

$$\Pi_1 : -2x + ay + 4z = 2$$

$$\Pi_2 : x + by - 2z = -1$$

$$\Pi_3 : 2x - y + cz = 3$$

Where $a, b, c \in \mathbb{R}$.

1. State the values of a, b and c for which
    1. The system has infinite solutions.
    2. The system is inconsistent. [7]
2. In the case where the system has infinite solutions, describe the geometric relationship between the three planes. [2]
3. In the case where the system is inconsistent, identify one of the geometric relationships that could exist between the three planes. [2]

Markscheme

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