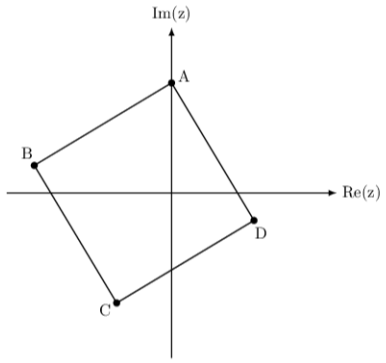
[Maximum mark: 6]
Solve the equation $z^3 = 1$, giving your answers in Cartesian form.

[Maximum mark: 4]
On the Argand diagram below, the point A represents the complex number $4\mathrm{i}$ and the point B represents the complex number $-5 + \mathrm{i}$. The shape ABCD is a square.



Determine the complex number represented by:

1. the point C; [2]
2. the point D. [2]

[Maximum mark: 6]

Consider the complex number $z = \frac{w_1}{w_2}$ where $w_1 = \sqrt{2} + \sqrt{6}\mathrm{i}$ and $w_2 = 3 + \sqrt{3}\mathrm{i}$.

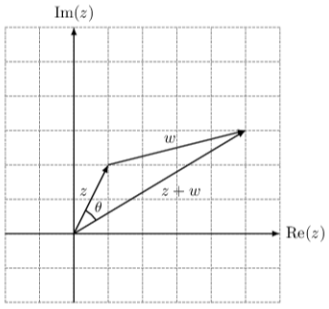
1. Express w_1 and w_2 in modulus-argument form and write down
    1. the modulus of z;
    2. the argument of z. [4]
2. Find the smallest positive integer value of n such that z^n is a real number. [2]

[Maximum mark: 6]

In this question give all angles in radians.

Let $z = 1 + 2\mathrm{i}$ and $w = 4 + \mathrm{i}$.

1. Find $z + w$. [1]
2. Find:
   1. $|z + w|$;
   2. $\arg(z + w)$. [3]
3. Find θ, the angle shown on the diagram below. [2]

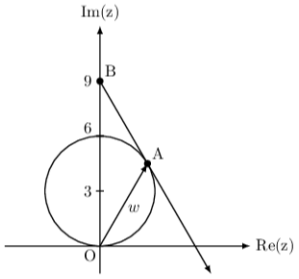


[Maximum mark: 8]
Let $z = 2 + \mathrm{i}$ and $w = 1 - 2\mathrm{i}$.

1. Find zw. [2]
2. Illustrate z, w and zw on the same Argand diagram. [3]
3. Let θ be the angle between zw and w. Find θ, giving your answer in radians. [3]

[Maximum mark: 6]

A circle of radius 3 and centre (0,3) is drawn on an Argand diagram. The tangent to the circle from the point $\mathrm{B}(0, 9)$ meets the circle at the point A as shown. Let $w = \overrightarrow{\mathrm{OA}}$.



1.
Show that $|w| = 3\sqrt{3}$. [2]

2.
Find $\arg w$. [2]

3.
Hence write w in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{R}$. [2]

[Maximum mark: 7]

The complex numbers w and z satisfy the equations

$$\frac{z}{w} = \mathrm{i},$$
$$w^* + 2z = 4 + 5\mathrm{i}.$$

Find w and z in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

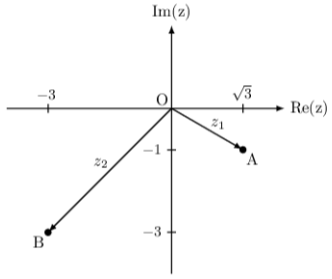
[Maximum mark: 6]

Consider the equation $\frac{3z}{5-z^*} = \mathrm{i}$, where $z = x + \mathrm{i}y$ and $x, y \in \mathbb{R}$.

Find the value of x and the value of y.

[Maximum mark: 7]

Points A and B represent the complex numbers $z_1 = \sqrt{3} - \mathrm{i}$ and $z_2 = -3 - 3\mathrm{i}$ as shown on the Argand diagram below.

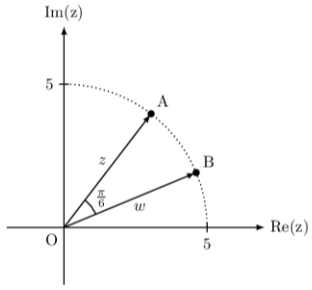


1.
Find the angle AOB. [3]

2.
Find the argument of $z_1 z_2$. [1]

3.
Given that the real powers of $pz_1 z_2$, for $p > 0$, all lie on a unit circle centred at the origin, find the exact value of p. [3]

[Maximum mark: 6]
The complex numbers z and w correspond to the points A and B as shown on the diagram below.



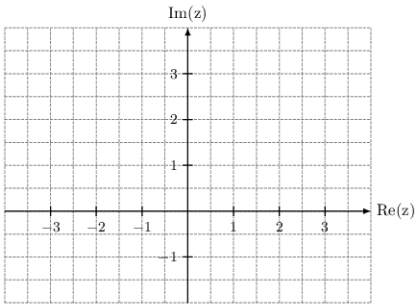
1.
Find the exact value of $|z - w|$. [2]
2.
    1.
    Find the exact perimeter of triangle AOB.
    2.
    Find the exact area of triangle AOB. [4]

[Maximum mark: 6]
Let $z = re^{i\frac{\pi}{3}}$ where $r \in \mathbb{R}^+$.

1.
For $r = \sqrt{2}$,

1.
express z^2 and z^3 in the form $a + bi$ where $a, b \in \mathbb{R}$;

2.
draw z^2 and z^3 on the following Argand diagram. [4]



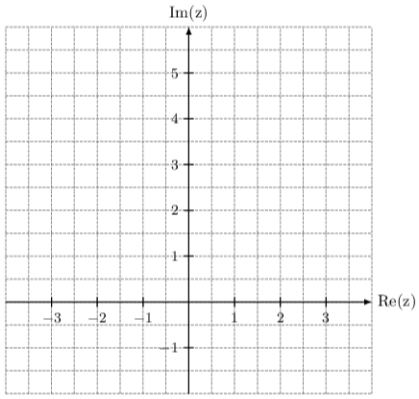
2.
Given that the integer powers of $w = (3 - 3i)z$ lie on a unit circle centred at the origin, find the value of r. [2]

[Maximum mark: 6]
Let $z = re^{i\frac{\pi}{6}}$ where $r \in \mathbb{R}^+$.

1.
For $r = \sqrt{3}$,

1.
express z^2 and z^3 in the form $a + bi$ where $a, b \in \mathbb{R}$;

2.
draw z^2 and z^3 on the following Argand diagram. [4]



2.
Given that the integer powers of $w = \dfrac{z}{6 + 2i}$ lie on a unit circle centred at the origin, find the value of r. [2]

[Maximum mark: 7]
Consider the complex numbers $u = 1 + 2\mathrm{i}$ and $v = 2 + \mathrm{i}$.

1.
Given that $\frac{1}{u} + \frac{1}{v} = \frac{6\sqrt{2}}{w}$, express w in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{R}$. [4]

2.
Find w^* and express it in the form $re^{i\theta}$. [3]

[Maximum mark: 9]

1.
Find three distinct roots of the equation $z^3 + 64 = 0$, $z \in \mathbb{C}$, giving your answers in modulus-argument form. [6]

The roots are represented by the vertices of a triangle in an Argand diagram.

1. Show that the area of the triangle is $12\sqrt{3}$. [3]

[Maximum mark: 6]
Let $z = 2\operatorname{cis} 2\theta$ where $0 < \theta < 45^\circ$. Find the modulus and argument of $z + 2$, expressing your answers in terms of θ.

[Maximum mark: 8]
Let $w = 2e^{i\frac{2\pi}{3}}$.

1.
    1.
    Write w, w^2 and w^3 in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{R}$.
    2.
    Draw w, w^2 and w^3 on an Argand diagram. [6]
2.
Find the smallest integer $k > 3$ such that w^k is a real number. [2]

[Maximum mark: 7]
Consider the equation $2z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$. Two of the roots of the equation are $\log_2 10$ and $\mathrm{i}\sqrt{5}$ and the sum of all the roots is $4 + \log_2 5$.
Show that $15a + d + 90 = 0$.

[Maximum mark: 12]
Consider the complex numbers $z_1 = 3\,\text{cis}(120^\circ)$ and $z_2 = 2 + 2\text{i}$.

1.
Calculate $\frac{z_1}{z_2}$ giving your answer both in modulus-argument form and Cartesian form. [7]

2.
Use your results from part (a) to find the exact value of $\sin 15^\circ \cdot \sin 45^\circ \cdot \sin 75^\circ$, giving your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$. [5]

[Maximum mark: 18]

1.
Express $-4 + 4\sqrt{3}\text{i}$ in the form $re^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [5]

Let the roots of the equation $z^3 = -4 + 4\sqrt{3}\text{i}$ be z_1, z_2 and z_3.

1.
Find z_1, z_2 and z_3 expressing your answers in the form $re^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [5]

On an Argand diagram, z_1, z_2 and z_3 are represented by the points A, B and C, respectively.

1.
Find the area of the triangle ABC. [4]

2.
By considering the sum of the roots z_1, z_2 and z_3, show that

$$\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right) = 0$$

[4]

[Maximum mark: 15]
Consider $w = \dfrac{z-1}{z+\mathrm{i}}$ where $z = x + \mathrm{i}y$ and $\mathrm{i} = \sqrt{-1}$.

1. If $z = \mathrm{i}$,
    1. write w in the form $r\,\mathrm{cis}\,\theta$;
    2. find the value of w^{14}. [5]
2. Show that in general,
$$w = \frac{(x^2 - x + y^2 + y) + \mathrm{i}(y - x + 1)}{x^2 + (y+1)^2}$$
[4]
3. Find condition under which $\mathrm{Re}(w) = 1$. [2]
4. State condition under which w is:
    1. real;
    2. purely imaginary. [2]
5. Find the modulus of z given that $\arg w = \dfrac{\pi}{4}$. [2]

[Maximum mark: 6]
On an Argand diagram, the complex numbers $z_1 = 2 + 2\sqrt{3}\mathrm{i}$, $z_2 = 1 - \mathrm{i}$ and $z_3 = z_1 z_2$ are represented by the vertices of a triangle.
The exact area of the triangle can be expressed in the form $p + \sqrt{q}$. Find the value of p and of q.

[Maximum mark: 19]

1.

1.
Expand $(\cos\theta + \mathrm{i}\sin\theta)^4$ by using the binomial theorem.

2.
Hence use de Moivre's theorem to prove that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$

3.
State a similar expression for $\sin 4\theta$ in terms of $\cos\theta$ and $\sin\theta$. [6]

Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where α is measured in degrees, be the solution of $z^4 - \mathrm{i} = 0$ which has the smallest positive argument.

1.
Find the modulus and argument of z. [4]

2.
Use (a) (ii) and your answer from (b) to show that $8\cos^4\alpha - 8\cos^2\alpha + 1 = 0$. [4]

3.
Hence express $\cos 22.5^\circ$ in the form $\dfrac{\sqrt{a+b\sqrt{c}}}{d}$ where $a, b, c, d \in \mathbb{Z}$. [5]

[Maximum mark: 19]

Let $z = \cos\theta + \mathrm{i}\sin\theta$, for $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$.

1.
   1.
   Find z^3 using the binomial theorem.
   2.
   Use de Moivre's theorem to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. [8]
2.
Hence show that $\frac{\sin 3\theta - \sin\theta}{\cos 3\theta + \cos\theta} = \tan\theta$. [6]
3.
Given that $\sin\theta = \frac{1}{3}$, find the exact value of $\tan 3\theta$. [5]

[Maximum mark: 22]

1.
Solve $2\sin(x + 120^\circ) = \sqrt{3}\cos(x + 60^\circ)$, for $x \in [0, 180^\circ]$. [5]
2.
Show that $\sin 75^\circ + \cos 75^\circ = \frac{\sqrt{6}}{2}$. [3]
3.
Let $z = \sin 4\theta + \mathrm{i}(1 - \cos 4\theta)$, for $z \in \mathbb{C}$, $\theta \in [0, 90^\circ]$.
   1.
   Find the modulus and argument of z in terms of θ.
   2.
   Hence find the fourth roots of z in modulus-argument form. [14]

[Maximum mark: 17]

1.
Solve the equation $z^3 = 27$, $z \in \mathbb{C}$, giving your answer in the form $z = r(\cos\theta + \mathrm{i}\sin\theta)$ and in the form $z = a + b\mathrm{i}$ where $a, b \in \mathbb{R}$. [6]

2.
Consider the complex numbers $z_1 = -1 + \mathrm{i}$ and $z_2 = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right)\right]$.

1.
Write z_1 in the form $r(\cos\theta + \mathrm{i}\sin\theta)$.

2.
Calculate $z_1 z_2$ and write in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{R}$.

3.
Hence find the value of $\tan\left(\frac{\pi}{12}\right)$ in the form $c + d\sqrt{3}$ where $c, d \in \mathbb{Z}$.

4.
Find the smallest $p \in \mathbb{Q}^+$ such that $(z_1 z_2)^p$ is a positive real number. [11]

[Maximum mark: 16]

1. Find the roots of $z^{16} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answer in the form $re^{i\theta}$, where $r, \theta \in \mathbb{R}^+$. [5]
2. Let S be the sum of the roots found in part (a).
   1. Show that $\mathrm{Re}(S) = \mathrm{Im}(S)$.
   2. By writing $\frac{\pi}{8}$ as $\frac{1}{2} \cdot \frac{\pi}{4}$, find the value of $\cos\left(\frac{\pi}{8}\right)$ in the form $\frac{\sqrt{a+\sqrt{b}}}{c}$, where a, b and c are integers to be determined.
   3. Hence, or otherwise, show that $S = \frac{1}{2}\left(\sqrt{2+\sqrt{2}} + \sqrt{2} + \sqrt{2-\sqrt{2}}\right)(1+\mathrm{i})$. [11]

[Maximum mark: 20]

1. Solve the equation $\sin(x + 90^\circ) = 2\cos(x - 60^\circ), 0^\circ < x < 360^\circ$. [5]
2. Show that $\sin 15^\circ + \cos 15^\circ = \frac{\sqrt{6}}{2}$. [4]
3. Let $z = 1 - \cos 4\theta - \mathrm{i} \sin 4\theta$, for $z \in \mathbb{C}, 0 < \theta < \frac{\pi}{2}$.
   1. Find the modulus and argument of z. Express each answer in its simplest form.
   2. Hence find the fourth roots of z in modulus-argument form. [11]

[Maximum mark: 21]

1.
Use de Moivre's theorem to find the value of $\left[\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right)\right]^{12}$. [2]

2.
Use mathematical induction to prove that

$$(\cos\alpha - \mathrm{i}\sin\alpha)^n = \cos(n\alpha) - \mathrm{i}\sin(n\alpha) \quad \text{for all } n \in \mathbb{Z}^+.$$

[6]

Let $w = \cos\alpha + \mathrm{i}\sin\alpha$.

1.
Find an expression in terms of α for $w^n - (w^*)^n$, $n \in \mathbb{Z}^+$, where w^* is the complex conjugate of w. [2]

2.
1.
Show that $ww^* = 1$.

2.
Write down and simplify the binomial expansion of $(w - w^*)^3$ in terms of w and w^*.

3.
Hence show that $\sin(3\alpha) = 3\sin\alpha - 4\sin^3\alpha$. [5]

3.
Hence solve $4\sin^3\alpha + (2\cos\alpha - 3)\sin\alpha = 0$ for $0 \leq \alpha \leq \pi$. [6]

Markscheme

Sorry.. please check revisiontown for markschemes. It was really hard scraping each dialog box and scaling correctly. Couldnt figure it out...