[Maximum mark: 6]
Given that $(x-4)$ is a factor of $f(x)=x^3-2x^2+ax+b$ and that division of $f(x)$ by $(x+2)$ leaves a remainder of 18, find the value of a and the value of b.

[Maximum mark: 5]
Let $f(x)=x^4+5x^3+ax^2+bx+2$, for $x\in\mathbb{R}$, where a, b are constants. The remainder when $f(x)$ is divided by $(x-1)$ is 6, and the remainder when $f(x)$ is divided by $(x+2)$ is -6. Find the value of a and the value of b.

[Maximum mark: 6]
The quadratic equation $x^2-kx+(k-3)=0$ has roots α and β such that $\alpha^2+\beta^2=6$. Without solving the equation, find the possible values of the real number k.

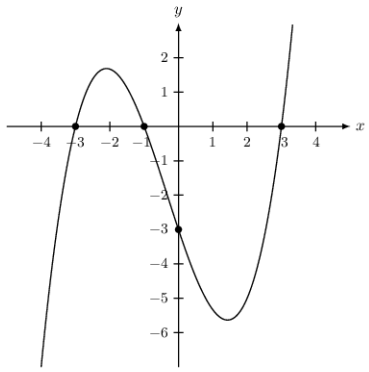
[Maximum mark: 5]
The quadratic equation $x^2 - kx + (k-1) = 0$ has roots α and β. Without solving the equation, find the possible values of the real number k given that $\alpha^2 + \beta^2 = 17$.

[Maximum mark: 5]
When $p(x) = x^2 - 2x + p$ is divided by $x - r$, the remainder is 4.
Given that $p, r \in \mathbb{R}$, find the largest possible value for p.

[Maximum mark: 5]
The polynomial $p(x) = 2x^4 + a_3x^3 + a_2x^2 + a_1x - 12$ is divisible by each of $(x+1)$, $(x-1)$ and $(x-2)$.
Find the values of a_1, a_2 and a_3.

[Maximum mark: 7]

Let $f(x) = ax^3 + bx^2 + cx + d$, for $x \in \mathbb{R}$, where $a, b, c, d \in \mathbb{Q}$. The diagram below shows part of the graph of $y = f(x)$.



1.

Using the information shown in the diagram, find the values of a, b, c and d. [4]

2.

Let $g(x) = -\frac{3}{4}f(-2x + 1)$.

1.

Find the coordinates of the points where the graph of $y = g(x)$ intercepts the x-axis.

2.

Find the y-intercept of the graph of $y = g(x)$. [3]

[Maximum mark: 7]
Consider the equation $32x^3 - 144x^2 + 214x - 105 = 0$.

1.
Find the sum and product of the roots of this equation. [1]
2.
The roots of this equation are three consecutive terms of an arithmetic sequence. Taking the roots to be $a - d, a, a + d$, solve the equation. [6]

[Maximum mark: 12]
Consider the polynomial $p(z) = z^5 + z^4 - z^3 + z^2 + 4z + 2$, for $z \in \mathbb{C}$.

1.
Write down the sum and product of the roots of $p(z) = 0$. [2]
2.
Show that $(z + 1)$ is a factor of $p(z)$. [2]

The polynomial can be written in the form $p(z) = (z + 1)^3(z^2 + cz + d)$.

1.
Find the value of c and the value of d. [5]
2.
Hence find the complex roots of $p(z) = 0$. [3]

[Maximum mark: 5]
When $p(x) = x^2 - 2x - c$ is divided by $(x - r)$, the remainder is -5.
Given that $c, r \in \mathbb{R}$, find the smallest possible value for c.

[Maximum mark: 7]
Two distinct roots for the polynomial equation $z^4 - 10z^3 + cz^2 + dz + 170$ are $a + \mathrm{i}$ and $1 + \mathrm{i}b$ where $a, b, c, d \in \mathbb{Z}$, $b > 0$.

1.
Write down the other two roots in terms of a and b. [1]

2.
Find the value of a and the value of b. [6]

[Maximum mark: 7]
Consider the equation $2z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$. Two of the roots of the equation are $\log_2 10$ and $\mathrm{i}\sqrt{5}$ and the sum of all the roots is $4 + \log_2 5$.
Show that $15a + d + 90 = 0$.

[Maximum mark: 8]

Let $p(x) = \frac{1}{4}x^5 - 2x^4 - 5x^3 + 40x^2 + 16x - 128$, for $x \in \mathbb{R}$.

1. For the polynomial equation $p(x) = 0$, state:
   1. the sum of the roots;
   2. the product of the roots. [3]

A new polynomial is defined by $q(x) = p(2x - 2)$.

1. For the polynomial equation $q(x) = 0$, find:
   1. the sum of the roots;
   2. the product of the roots. [5]

[Maximum mark: 13]

It is given that $f(x) = 2x^4 + 5x^3 + ax^2 + bx + 4$, for $x \in \mathbb{R}$, where $a, b \in \mathbb{Z}^+$.

1. Given that $x^2 + x - 2$ is a factor of $f(x)$, find the values of a and b. [4]
2. Factorise $f(x)$ into a product of linear factors. [4]
3. Sketch the graph of $y = f(x)$, labeling the maximum and minimum points and the x and y intercepts. [3]
4. Using your graph, state the range of values of c for which $f(x) = c$ has exactly four distinct real roots. [2]

[Maximum mark: 14]
The cubic polynomial equation $x^3 + bx^2 + cx + d = 0$ has three roots x_1, x_2 and x_3. By expanding the product $(x - x_1)(x - x_2)(x - x_3)$, show that

1.
    1.
    $b = -(x_1 + x_2 + x_3)$;
    2.
    $c = x_1x_2 + x_1x_3 + x_2x_3$;
    3.
    $d = -x_1x_2x_3$. [3]

It is given that $b = -9$ and $c = 45$ for parts (b) and (c) below.

1.
    1.
    In the case that the three roots x_1, x_2 and x_3 form an arithmetic sequence, show that one of the roots is 3.
    2.
    Hence determine the value of d. [5]
2.
In another case the three roots form a geometric sequence. Determine the value of d. [6]

Markscheme

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