[Maximum mark: 5]

1.
Sketch the graph of $y = (x-2)^2 - 4|x-2| - 1$, for $-3 \leq x \leq 7$. [3]

2.
Hence, or otherwise, solve the equation $(x-2)^2 - 4|x-2| - 1 = 0$. [2]

[Maximum mark: 6]

Consider the function $f(x) = \frac{e^x - e^{-x}}{2}$, $x \in \mathbb{R}$.

[2]

1. Show that f is an odd function.

Now, consider the function g given by $g(x) = \frac{x^4 + 2}{2x}$, $x \in \mathbb{R}, x \neq 0$.

[4]

1. By considering the graph of $y = f(x) - g(x)$, solve $f(x) > g(x)$ for $x \in \mathbb{R}$.

[Maximum mark: 8]
A population of endangered snow leopards, P, can be modelled by the equation

$$P_t = P_0 e^{kt},$$

where P_0 is the initial population, and t is measured in years.

1.
After one year, it is estimated that $\frac{P_1}{P_0} = 0.93$.
    1.
    Find the value of k.
    2.
    Interpret the meaning of the value of k. [3]
2.
Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.50$. [5]

[Maximum mark: 5]
Consider the graphs of $y = -\frac{1}{2}|x|$ and $y = 2|x| - a$, where $a \in \mathbb{Z}^+$.

1.
Sketch the graphs on the same set of axes. [2]
2.
Given that the graphs enclose a region of area 40 square units,
find the value of a. [3]

[Maximum mark: 8]
Consider the function $f(x) = x^2 \arcsin(x)$, for $-1 \leq x \leq 1$.

1. Sketch the graph of $y = f(x)$. [3]
2. Write down the range of f. [2]
3. Solve the inequality $\left|x^2 \arcsin(x)\right| > 0.5$. [3]

[Maximum mark: 7]
Let $f(x) = \dfrac{9 - 12x}{cx - 20}$, for $x \neq \dfrac{20}{c}$, where $c \neq 0$.

1. The line $x = 5$ is a vertical asymptote to the graph of $y = f(x)$.
    1. Find the value of c.
    2. Write down the equation of the horizontal asymptote to the graph of $y = f(x)$. [4]
2. The line $y = h$, where $h \in \mathbb{R}$, intersects the graph of $y = \left|f(x)\right|$ at exactly one point. Find the possible values of h. [3]

[Maximum mark: 17]

The function f is defined by $f(x) = 1 + \frac{4x}{x+3}$, for $x \neq -3$.

1.
Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

2.
Find an expression for f^{-1}. [4]

3.
Find all values of x for which $f(x) = f^{-1}(x)$. [3]

4.
Solve the inequality $|f(x)| < 2$. [4]

5.
Solve the inequality $f(|x|) < 2$. [2]

[Maximum mark: 8]

1.
Sketch the curve $y = -\left|\frac{5}{x-2}\right|$ and line $y = -x - 4$ on the same axes, clearly indicating any x and y intercepts and any asymptotes. [3]

2.
Find the exact solutions to the equation $x + 4 = \frac{5}{|x-2|}$. [5]

[Maximum mark: 8]

1.
Solve the inequality $x^2 \geq 2x + 3$. [2]

2.
Use mathematical induction to prove that $2^n > n^2 - 2$ for all $n \in \mathbb{Z}^+$, $n \geq 3$. [6]

[Maximum mark: 22]
Let $f(x) = x^4 - 0.4x^3 - 2.85x^2 + 0.9x + 1.35$, for $x \in \mathbb{R}$.

1.
Find the solutions for $f(x) > 0$. [3]
2.
For the graph of $y = f(x)$,
    1.
    find the coordinates of local minimum and maximum points.
    2.
    find the x-coordinates of the points of inflexion. [5]

The domain of f is now restricted to $[a, b]$ where $a, b \in \mathbb{R}^+$.

1.
    1.
    Write down the smallest value of $a > 0$ and the largest value of $b > 0$ for which f has an inverse. Give your answers correct to **three significant figures**.
    2.
    For these values of a and b, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing clearly the coordinates of the end points of each curve.
    3.
    Solve $f^{-1}(x) = 0.5$. [6]

Let $g(x) = \frac{2}{3}\sin(2x - 1) + \frac{1}{2}, \frac{1}{2} - \frac{\pi}{4} \leq x \leq \frac{1}{2} + \frac{\pi}{4}$.

1.
    1.
    Find an expression for g^{-1} and state its domain.
    2.
    Solve $(f^{-1} \circ g)(x) < 0.5$. [8]

[Maximum mark: 22]

Consider the function defined by $f(x) = (2x - 6)\ln(x + 3) + x$ for $x \in \mathbb{R}$, $x > p$.

1. Find the value of p. [2]
2. Find an expression for $f'(x)$. [3]

The graph of $y = f(x)$ has no points of inflexion. [3]

1. Determine if the graph of $y = f(x)$ is concave down or concave up over its domain.

The function g is defined by $g(x) = 3\ln\left(\frac{1}{x+3}\right) + x$, for $x \in \mathbb{R}$, $x > -3$.

1. Find an expression for $g'(x)$. [4]
2. Find the horizontal and vertical asymptotes of $g'(x)$. [2]
3. Find the exact value of the minimum of $y = g(x)$. [3]
4. Solve $f(x) < g(x)$ for $x \in \mathbb{R}$, $x > -3$. [5]

Markscheme

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